# Development of interferometric fringe disappearance method used to calibrate accelerometers

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# ABSTRACT

Nowadays, piezoelectric accelerometers are among the most developed transducers used for the measurement of mechanical vibrations due to its linearity, short and long term stability, size, and weight properties.

The fringe disappearance method classically has two experimental approaches, they two measure a different characteristic of the photodiode output which is at the interferometer. The first approach, called the  $J_0$  null-method, measures the DC output signal from the photodiode and, the second one, called the  $J_1$  null-method, measures the AC output signal from the photodiode using a band-pass filter tuned at the vibration frequency.

This paper shows a novel approach, which covers both classical methods is presented. In this approach is used a spectral analyzer with a cut-off frequency of 25 kHz, using this facility we can apply from  $J_0$  to  $J_9$  (Bessel functions of first kind), depending on the signal frequency applied by the vibration exciter. The novel approach proposed here has much more resolution that the two classical approaches.

Experimental results from interferometric calibrations are used to validate the obtained model of the irradiance light, which depends on the Bessel functions of first kind and *N*th order.

The fringe disappearance method, also known as Bessel function minimum-point method, is applied within the frequency range from 1 kHz to 5 kHz. A vibration exciter of electrodynamic type reproduces a simple harmonic motion of the accelerometer at a constant acceleration level; however due to the exciter's suspension the sinusoidal movement shows a noisy rocking motion, this rocking motion produces distortions at the output signals on both the laser interferometer (reference) and the accelerometer (test). The mechanical distortion produced by the vibration exciter is measured and its effects on the uncertainty are analyzed. An alignment of the Michelson interferometer which is less sensitive to the tilting is proposed.

# 1. INTRODUCTION

In the field of mechanical vibrations, among the main aims of accurate measurements is to have a better understanding of the dynamic performance of a certain device under study, then through a mathematical model we can predict the device's dynamic behavior under stated conditions. One way to obtain the best accuracy in field applications is calibrating the transducers, which will be used, by absolute calibration methods, i.e. laser interferometry.

The high accuracy obtained in absolute calibration methods is transferred to a transducer, which will be used directly in field applications. Piezoelectric accelerometers are preferred in mechanical vibrations applications because of its linearity, short and long term stability, size, and weight properties.

Several methods can be used in laser interferometry, the fringe disappearance method, also known as Bessel function minimum-point method, is applied in the measurement of mechanical vibrations within the frequency range from 1 kHz to 5 kHz. The sensitivity of an accelerometer can be estimated using the fringe disappearance method in this frequency range [3].

The fringe counting method has been used for several years, but in a limited way. By this limited approach, the displacement magnitude can be determined at fixed points, where zero crossings of the Bessel functions occur and is known as fringe disappearance points. But up to now only the  $J_0$  and  $J_1$  are used, the Bessel functions of first kind and order zero and one, respectively.

The novel approach showed in this work, covers both classical approaches but goes further because it can even be used from  $J_0$  to  $J_N$ , where *N* means both the Bessel function order and the *N*th harmonic of the simple harmonic motion used to generate the mechanical vibration. If a FFT analyzer is connected to the photodiode output, the Nth harmonics can be seen in the frequency space up to the cut off frequency of the analyzer.

Laser Doppler interferometry basics are discussed in the following section, highlighting the conceptual frame of the fringe disappearance method.

#### 2. LASER INTERFEROMETRY THEORY

The experimental arrangement of the Michelson interferometer used to calibrate accelerometers is shown in Figure 1.



Figure 1. Michelson interferometer used to calibrate accelerometers

In this work the magnetic field is not considered because the photodiode does not sense it. Therefore, only the electric fields will be considered here. In the Michelson interferometer, the reference beam,  $E_1$ , can be expressed as,

$$E_1 = A_1 \exp j \left( wt + j_1 \right) \tag{1}$$

where  $A_1$  is the electric field intensity,  $\omega$  is the reference circular frequency and,  $\varphi_1$  is the optical phase. On the other hand, the measuring beam is first transmitted through the beam splitter and then reflected at the measuring surface, when the beam comes back from the measuring surface it can be expressed as,

$$E_2 = A_2 \exp j \left( \mathbf{w}' t \pm \mathbf{j}_2 \right) \tag{2}$$

where  $A_2$  is the electric field intensity,  $w' = w/(1 \pm v/c)$ , is the shifted frequency due to the Doppler effect, v is the target instantaneous velocity, c is the speed of light and,  $\varphi_2$  is the optical phase of the measuring beam. Cloud [1995] shows a more detailed derivation of laser Doppler interferometry. The two beams combine to each other at the beam splitter, then they are directed to the photodiode which measures the irradiance. The output irradiance of the interferometer can be expressed as,

$$I = |E_{1} + E_{2}|^{2}$$

$$I = A_{1}^{2} \exp 2j(\mathbf{w}t + \mathbf{j}_{1}) + A_{2}^{2} \exp 2j(\mathbf{w}'t + \mathbf{j}_{2})$$

$$+ 2A_{1}A_{2} \exp j[(\mathbf{w} + \mathbf{w}')t + (\mathbf{j}_{1} + \mathbf{j}_{2})]$$

$$+ 2A_{1}A_{2} \exp j[(\mathbf{w} - \mathbf{w}')t + (\mathbf{j}_{1} - \mathbf{j}_{2})]$$
(3)

The firs three terms in equation 3 have frequencies higher than  $\omega$ , the beam propagation exceeds the speed of light and therefore the medium, which in this case is air, become dissipative. Therefore the irradiance, *I*, is proportional to the experimental measurement by the photodiode and can be expressed in a simpler form, as follows,

$$I = 2A \exp j \left( \mathbf{w}_B t - \mathbf{j} \right) \tag{4}$$

where,  $\omega_{\text{B}}{=}\omega{-}\omega'$ , is beat frequency and, j=[d+2x(t)]2p/l, is the optical phase;  $\lambda$ , is the He-Ne wavelength;  $\delta$ , is the path difference between the two beams;  $\xi(t){=}\xi{sin}(2\pi ft)$ , is the simple harmonic motion described by the accelerometer (and moving mirror), with a displacement amplitude,  $\xi$ , and a constant frequency, f.

From the above discussion, we can conclude three important facts: i. the optical phase,  $\varphi$ , depends upon the displacement,  $\xi(t)$ ; ii. the beat frequency,  $\omega_B$ , is proportional to the instantaneous velocity of the accelerometer and; iii. the frequencies,  $\omega$  and  $\omega'$ , are close enough so that their difference is a measurable frequency by an ordinary photodiode. Silva [1999] gives further analysis about laser interferometry and the measurement of optical phase.

## 3. FRINGE DISAPPEARANCE METHOD

The accuracy of the fringe disappearance method depends on the displacement stability of the mechanical vibration to which the accelerometer is joined. The accelerometer is mounted using a constant torque of 2 N·m. The simple harmonic motion is exerted by the vibration exciter and should be linear, it means that lateral motions should be practically negligible. The irrandiance expression of equation 4, is shown in Figure 2.





The irradiance, obtained from the photodiode output (see figure 2b), is displayed in the frequency domain and, it generates a number of harmonic components with a frequency span equal to the excitation frequency applied to the accelerometer. Experimentally, using a FFT analyzer we can see the frequency domain of the photodiode output. In order to expand the expression of the light irradiance, I(t), of equation 4 we should remember the Jacobi's series,

$$\cos(A\sin B) = J_0(A) + 2J_2(A)\cos 2B + \cdots$$
 (5a)

and

Submitted to the SEM Annual Conference on Experimental Mechanics. Portland, Oregon, USA. June 4-6, 2001.  $sin(A sin B) = 2J_1(A) cos B + 2J_3(A) cos 3B + \cdots$  (5b)

Using equations 5, we find the expanded expression for the average light irradiance, I(t),

$$I = A + B \cos\left\{\frac{4pd}{l}\left[J_{0}\left(\frac{4px}{l}\right) + 2J_{2}\left(\frac{4px}{l}\right)\cos 2(2pft) + \cdots\right]\right\}$$
$$-B \sin\left\{\frac{4pd}{l}\left[J_{1}\left(\frac{4px}{l}\right)\cos(2pft) + 2J_{3}\left(\frac{4px}{l}\right)\cos 3(2pft) + \cdots\right]\right\}$$
(6)

The fringe disappearance method, which is based on the determination of displacement, uses the arguments corresponding to the zero crossings of the Bessel function of the first kind and nth order, see equation 6.

In order to experimentally determine certain Bessel function minimum points,  $J_N$ , where  $N=0,1,2,3,\ldots$ ; we should observe the frequency domain. When a FFT analyzer displays the power spectral density, PSD, of the light irradiance, it shows several frequency components. The first frequency component is displayed at the DC component, the second component is displayed at the excitation frequency, *f*, and the other frequency components are displayed at harmonics of the excitation frequency applied on the accelerometer.

The magnitude of each one of the frequency components, in the photodiode output, depends on the displacement magnitude of the accelerometer,  $\xi$ . This means that, keeping a constant excitation frequency, the displacement magnitude increases and at the displacements stated in Table 1, the PSD will show a minimum at the frequency that corresponds to one Bessel minimum, J<sub>N</sub>. This process is illustrated in figure 3.

	0	1	2	3
ξ [nm]	0	121	278	435
$J_0(4\mathbf{px}/\mathbf{l})$	1	2,4	5,5	8,7
ξ [nm]	0	193	353	512
$J_1(4\mathbf{px}/\mathbf{l})$	0	3,8	7,0	10,2
ξ [nm]	0	258	423	585
$J_2(4\mathbf{px}/\mathbf{l})$	0	5,1	8,4	11,6
ξ [nm]	0	321	491	655
$J_3(4\mathbf{px}/\mathbf{l})$	0	6,4	9,8	13,0
ξ [nm]	0	382	557	723
$J_4(4\mathbf{px}/\mathbf{l})$	0	7,6	11,1	14,4
ξ [nm]	0	441	621	790
$J_5(4\mathbf{px}/\mathbf{l})$	0	8,8	12,3	15,7

Table 1. Bessel function minimum points,  $J_{N}$ , N=0,1,2,3 and so on.

### 4. RESULTS

In the experimental application of the fringe disappearance method, the power spectral density of the photodiode output, which is the light irradiance, I(t), is proportional to the displacement magnitude,  $\xi$ , of the mechanical vibration applied on the accelerometer.

In the figure 3 are shown the results of the measurement when a sinusoidal motion,  $\xi(t)=\xi\sin(2\pi ft)$ , is applied on the accelerometer. The excitation frequency is kept constant during the measurement process, *f*=4 kHz and, the displacement magnitude,  $\xi$ , increases from zero to 321 nm.

When the displacement magnitude,  $\xi$ , is risen up to 121 nm, the first minimum point is presented in the  $J_0$  function. If we keep increasing the displacement magnitude, we will find the following minimum points following the sequence shown in figure 3. The order sequence ( $J_N$ ) goes from the smallest to the biggest values of the displacement magnitude,  $\xi$ .



Figure 3a. FFT when the displacement amplitude;  $\xi$ =121 nm and a frequency, f=4kHz.



Figure 3b. FFT when the displacement amplitude;  $\xi$ =193 nm and a frequency, f=4kHz.

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Figure 3b. FFT when the displacement amplitude;  $\xi$ =258 nm and a frequency, f=4kHz.



Figure 3b. FFT when the displacement amplitude;  $\xi$ =278 nm and a frequency, f=4kHz.



Figure 3a. FFT when the displacement amplitude;  $\xi$ =321 nm and a frequency, f=4kHz.

The upper limit of the displacement magnitude,  $\xi$ , which is applied on the accelerometer, depends on the dynamic limit of the electrodynamic vibration exciter which is being used. At high displacements are risen, the harmonic distortion increases and we can not suppose sinusoidal motion anymore.

In the case when it is not possible to use a FFT analyzer, then the minimum points can be found using a band pass filter tuned at the frequencies that correspond to every  $J_N$  Bessel function.

Submitted to the SEM Annual Conference on Experimental Mechanics. Portland, Oregon, USA. June 4-6, 2001. However, it is important to note that the sequence is always the same as the one shown in figure 3, it means, from the smallest to the biggest displacement magnitude applied on the accelerometer.

In the calibration of the sensitivity magnitude of an accelerometer using the fringe disappearance method, once the displacement magnitude has been found and, supposing a sinusoidal motion. Then, the acceleration function is found deriving twice the displacement function and, therefore, the acceleration amplitude, a, equals the displacement amplitude times the excitation frequency squared expressed in radians per second,

$$a = \mathbf{x} (f/2\mathbf{p})^2 \tag{7}$$

Finally the sensitivity magnitude, S, of the accelerometer is the output voltage of the charge amplifier connected to the accelerometer, V, divided by the acceleration magnitude, a, found in equation 7,

$$S = V/a \tag{8}$$

## 5. CONCLUSION

Using the novel approach of the fringe disappearance method shown in this paper, it is possible to increase the resolution of the measurement in comparison with both classical approaches, which only use  $J_0$  and  $J_1$ , respectively. For example, in the range of displacement magnitude from zero to 353 nm, both  $J_0$  and  $J_1$ , have only two minimum points each one; while the approach shown here considers six minimum points, three times more!.

Even the new ISO standards regarding the accelerometer calibration [3] only consider the J0 and J1 Bessel functions, therefore the method described here may lead to a further improvement of these International Standards.

If a smaller resolution is needed in the measurement of the displacement magnitude of the accelerometer, then it is necessary to use interferogram analysis and phase unwrapping. Those researchers interested on these topics may find useful information in Malacara [1998].

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